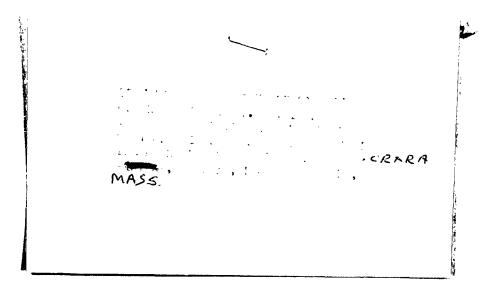
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Positioning of Ferrimagnets in a Resonant Cavity to Induce Magnetostatic Modes of Oscillation

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Abstract

Magnetostatic modes of oscillation arise when nonuniform magnetic fields are applied to a ferrimagnetic sample. This can be achieved by positioning a ferrimagnet within a microwave resonating cavity. The purpose of this paper is to find where in a cavity a sample must be placed to obtain particular modes of oscillation. Correct identification of magnetostatic modes is useful in computing one of the most important properties of magnetic substances—saturation magnetization.

Two approaches are taken to the foregoing problem. The first yields a general equation for the location which will produce a desired mode. This equation tends to be mathematically complicated. The second approach places a sample in a symmetrical field position within the resonant cavity, checking to see which modes are present and their relative intensities. The latter method is mathematically simpler and permits practical application.

Positioning of Ferrimagnets in a Resonant Cavity to Induce Magnetostatic Modes of Oscillation

1. INTRODUCTION

If a ferrimagnetic sample is placed in an oscillating microwave field and is subjected to an appropriately oriented d-c magnetic field (H_{dc}) , it will experience certain natural resonant fields on its surface. These field configurations are referred to as the natural magnetostatic modes of oscillation, and expressed in three parameters (n, m, r). The letters (n, m, r) arise from the notation used in the Legendre polynomials which express solutions of field resonances on spherical surfaces. A physical understanding of "n" and "m" is possible by studying the locus of $\begin{bmatrix} P_m & (\cos \theta) \\ \sin & \phi \end{bmatrix} \begin{bmatrix} \cos m \phi \\ \sin & \phi \end{bmatrix} = 0$ on a sphere with center at origin. This will divide the sphere into (n-m) parallels of latitude symmetrical about the equator $(\theta = 90^{\circ})$ and of m great circles through the pole, two consecutive ones being inclined at an angle π/m to one another. The field configuration for the (4,3,0) mode has been outlined by Walker and may be used as an example of the type pattern set up in a sphere.

The resonant fields on a sphere can be expressed in terms of a scalar magnetic potential ψ given by the P.D.E.:

$$(1+K)\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right] + \frac{\partial^2 \psi}{\partial z^2} = 0.$$
 (1)

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Walker has obtained this equation by solving Maxwell's equations (div B = 0, curl H = 0) and the equation of motion $\frac{dM}{dt} = \Gamma(M \times H)$ simultaneously. Fletcher and Bell have solved for ψ in the above P.D.E. for the case of a sphere. In their paper they have tabulated the general $\psi \frac{m}{n}$ for n = m + s with s = 0 to 5 and have also listed particular $\psi \frac{m}{n}$ for n = 0 to 5.

2. PROCEDURE

In order to find the location inside the cavity that will produce a desired resonance configuration, the following boundary conditions must be satisfied. First, the tangentially directed gradient of the magnetic scalar potential associated with the desired mode must be equated to the tangential component of the r-f magnetic field of the cavity. Secondly, the radial component of H_{rf} plus magnetization is equated to the radial H_{rf} in the cavity. These conditions are more conveniently expressed in the following form:

(Hφ) | outside =
$$(\nabla \psi)$$
 | inside = (Hφ) inside r = a r = a (2a)

$$(H_r)$$
 | outside = $[1 + K \sin^2 \theta] \left(\frac{\partial \psi}{\partial r} \operatorname{int}\right)_{r=a}$

$$+ \frac{K}{a} \sin \theta \cos \theta \left(\frac{\partial \psi}{\partial \theta} \right)_{r=a}^{int} - \frac{j\nu}{a} \left(\frac{\partial \psi}{\partial \phi} \right)_{r=a}^{int}$$
 (2b)

When these boundary conditions on the sphere are matched, the sample is located in the correct position within the cavity.

A detailed analysis of this procedure is carried out for the (2,2,0) mode in the Appendix. According to this method, it is theoretically possible to determine the location in the cavity that will produce specified modes of oscillation on the sphere. One merely looks up the $\psi^{\,m}_{\,n}$ for the desired mode in Fletcher and Bell⁵ and matches boundary conditions as described in Eqs. (2g) and (2b) on the surface of the sphere. Although mathematically this method will produce the desired results, the practical application may result in computational difficulties.

For this reason, it is often more convenient to assume a position within the cavity and work backwards to see what modes are present and their relative intensity. This is the method used by Fletcher, Solt, and Bell, ⁶ successful in achieving the following modes for various sample positions:

By placing the sample in a position symmetrical with the r-f field within the resonant cavity and changing the orientation of the H_{dc} field, they were able to observe the above modes and tabulate their calculated and observed intensities. A comprehensive bibliography follows the Appendix.

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Appendix

POSITION CALCULATION FOR (2, 2, 0) MODE

The resonant microwave field configurations for the microwave ${\rm TE}_{20\,1}$ mode are:

$$\mathbf{E}_{\mathbf{x}^{\mathbf{i}}} = \mathbf{E}_{\mathbf{z}^{\mathbf{i}}} = \mathbf{H}_{\mathbf{y}^{\mathbf{i}}} = 0 \tag{3a}$$

$$E_{y^i} = \eta 2 \frac{(2\delta)}{\lambda} A \sin \frac{2\pi x^i}{\delta} \sin \frac{1\pi z^i}{d}$$
 (3b)

$$H_{x'} = -j \frac{2(2\delta)}{\lambda} A \sqrt{1 - (\frac{\lambda}{2\delta})^2} \sin \frac{2\pi x'}{\delta} \cos \frac{1\pi z'}{d}$$
 (3c)

$$H_{z'} = jA \cos \frac{2\pi x'}{\delta} \sin \frac{1\pi z'}{d}$$
 (3d)

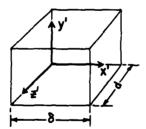


Figure 1. Rectangular Cavity with Superimposed Coordinate System

The above field representations can be expressed in another rectangular coordinate system if the following transformations are made:

$$x' = y_1, y' = z_1, z' = x_1$$
 (4g, b, c)

Then Eqs. (3) can be expressed as follows:

$$E_{y_1} = E_{x_1} = H_{z_1} = 0$$
 (5a)

$$E_{z_1} = \eta 2 \frac{(2\delta)}{\lambda} A \sin \frac{2\pi y_1}{\delta} \sin \frac{1\pi x_1}{d}$$
 (5b)

$$H_{y_1} = -j \frac{2(2\delta)}{\lambda} A \sqrt{1 - (\frac{\lambda}{2\delta})^2} \sin \frac{2\pi y_1}{\delta} \cos \frac{1\pi x_1}{d}$$
 (5c)

$$H_{x_1} = jA \cos \frac{2\pi y_1}{\delta} \sin \frac{1\pi x_1}{d}$$
 (5d)

[If the TE_{101} mode is desired, replace δ by 2δ in the above equations.] The scalar potential for the (2, 2, 0) magnetostatic mode is given by Fletcher and Bell^3 as:

$$\psi_2^2 = \frac{9}{-a^2 K} \left[G_2^2 (x^2 - y^2) + jH_2^2 2xy \right]$$
 (6a)

where

$$G_2^2 = \frac{a^2 Z_2^2(5)}{P_2^2(\xi_0)} \left[A_2^2 \left(n + 1 + \frac{\xi_0 P_2^{2'}(\xi_0)}{P_2^2(\xi_0)} \right) - \nu 2B_2^2 \right]$$
 (6b)

$$H_2^2 = \frac{a^2 Z_2^2(5)}{P_2^2(\xi_0)} \left[-\nu 2A_2^2 + B_2^2 \left(n + 1 + \frac{\xi_0 P_2^{2'}(\xi_0)}{P_2^2(\xi_0)} \right) \right]$$
 (6c)

$$Z_{2}^{2} = \frac{1}{\left(n+1+\frac{\xi_{0}P_{2}^{2'}(\xi_{0})}{P_{2}^{2}(\xi_{0})}\right)^{2}} - 4\nu^{2}$$
(6d)

$$n+1+\frac{\xi_{o}P_{2}^{2^{1}}(\xi_{o})}{P_{2}^{2}(\xi_{o})}=5+\frac{2\Omega_{H}}{\Omega_{H}^{2}-\Omega^{2}}$$
(6e)

$$K = \frac{\Omega_{\rm H}}{\Omega_{\rm H}^2 - \Omega^2} \tag{6\underline{t}}$$

$$v = \frac{\Omega}{\Omega_{\rm H}^2 - \Omega^2} \tag{6g}$$

$$\Omega_{\mathbf{H}} = \frac{\mathbf{H_i}}{4\pi\mathbf{M}} \tag{6h}$$

$$\Omega = \frac{\omega}{4\pi \Gamma M} \tag{6i}$$

$$H_{i} = H_{o} - \frac{4\pi M}{3}$$
 (6.j)

$$\xi_{O} = \sqrt{1 + 1/K} \tag{6k}$$

where A_2^2 and B_2^2 describe the amplitude of the applied r-f field. When $A_2^2 = B_2^2$, the ϕ dependence becomes $e^{2j\phi}$ and circular polarization occurs inside and outside the sample.

Define new variables

$$\eta_1 = \frac{9G_2^2}{-a^2K}$$
(7a)

$$n_2 = \frac{9H_2^2}{2K} \tag{7b}$$

$$\psi_2^2 = \eta_1(x^2 - y^2) + j\eta_2(2xy) \tag{7c}$$

Perform the following coordinate transformations:

$$x = r \cos \phi \sin \theta \tag{8a}$$

$$y = r \sin \phi \sin \theta \tag{8b}$$

$$z = r \cos \theta$$
 (8c)

$$\hat{i}_{x} = \hat{i}_{r} \cos \phi - \hat{i}_{\phi} \sin \phi \tag{8d}$$

$$\hat{\mathbf{i}}_{\mathbf{v}} = \hat{\mathbf{i}}_{\mathbf{r}} \sin \phi + \hat{\mathbf{i}}_{\mathbf{b}} \cos \phi \tag{8e}$$

where this coordinate system is referred to the center of the sphere. The scalar potential in spherical coordinates then becomes:

$$\psi_2^2 = \eta_1(r^2 \cos^2 \phi \sin^2 \theta - r^2 \sin^2 \phi \sin^2 \theta) + j\eta_2(2r^2 \cos \phi \sin \phi \sin^2 \theta)$$
 (9)

The tangential component of $H(H\phi)$ is the ϕ component of the gradient ψ from Eq. (9). This is found to be:

$$(H\phi)_{\text{inside}} = \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \phi} = \frac{-1}{r \sin \theta} \left[\eta_1 (r^2 2 \cos \sin \phi \sin^2 \theta + r^2 2 \sin \phi \cos \phi \sin^2 \theta) + j \eta_2 (2r^2 \sin^2 \phi \sin^2 \theta - 2r^2 \cos^2 \phi \sin^2 \theta) \right]$$

$$(10\underline{a})$$

Simplifying, we get:

$$(H\phi)_{\text{inside}} = -\eta_1(2r\sin 2\phi \sin \theta) + j\eta_2(2r\cos 2\phi \sin \theta)$$
 (10b)

When a change of coordinates

$$x_1 = x + x_a \tag{11a}$$

$$y_1 = y + y_g \tag{11b}$$

is applied to the Eqs. in (5) the $H_{\mathbf{rf}}$ field in the cavity is given by:

$$H = H_{x}\hat{i}_{x} + H_{y}\hat{i}_{y} = jA \cos \frac{2\pi}{\delta} (y_{g} + y) \sin \frac{1\pi}{d} (x_{g} + x) \hat{i}_{x}$$

$$-j \frac{2(2\delta)}{\lambda} A \sqrt{1 - (\frac{\lambda}{2\delta})^{2}} \sin \frac{2\pi}{\delta} (y_{g} + y) \cos \frac{1\pi}{d} (x_{g} + x) \hat{i}_{y}$$
(12)

where x_g and y_g are the distances from a corner of the cavity to the center of the sphere, and x and y are the coordinates of any point with respect to the center of the sphere.

A transformation of Eq. (12) into spherical coordinates using Eqs. (8) with the center of the sphere as the origin yields

$$\begin{split} \widetilde{H} &= -j \frac{2(2\delta)}{\lambda} A \sqrt{1 - (\frac{\lambda}{2\delta})^2} \sin \frac{2\pi}{\delta} (y_g + r \sin \theta \sin \phi) \\ &\cdot \cos \frac{1\pi}{d} (x_g + r \sin \theta \cos \phi) \cdot (\hat{i}_r \sin \phi + \hat{i}_{\phi} \cos \phi) \\ &+ j A \cos \frac{2\pi}{\delta} (y_g + r \sin \theta \sin \phi) \sin \frac{1\pi}{d} (x_g + r \sin \theta \cos \phi) \\ &\cdot (\hat{i}_r \cos \phi - \hat{i}_{\phi} \sin \phi) \end{split}$$
(13)

From Eq. (13), the Ho is found to be:

$$H\phi = -j \frac{2(2\delta)}{\lambda} A \sqrt{1 - (\frac{\lambda}{2\delta})^2} \sin \frac{2\pi}{\delta} (y_g + r \sin \theta \sin \phi)$$

$$\cos \frac{1\pi}{d} (x_g + r \sin \theta \cos \phi) \cos \phi$$

$$-jA \cos \frac{2\pi}{\delta} (y_g + r \sin \theta \sin \phi) \sin \frac{1\pi}{d} (x_g + r \sin \theta \cos \phi) \sin \phi \qquad (14)$$

Applying the boundary condition for the ϕ components of H on the surface of the sphere, we equate Eq. (10b) to Eq. (14), obtaining

$$-\eta_{1} \left(2a \sin 2\phi \sin \theta\right) + j\eta_{2} \left(2a \cos 2\phi \sin \theta\right) =$$

$$-j \frac{2(2\delta)}{\lambda} A \sqrt{1 - \left(\frac{\lambda}{2\delta}\right)^{2}} \sin \frac{2\pi}{\delta} \left(y_{g} + a \sin \theta \sin \phi\right)$$

$$\cos \frac{1\pi}{d} (x_{g} + a \sin \theta \cos \phi) \cos \phi$$

$$-jA \cos \frac{2\pi}{\delta} (y_{g} + a \sin \theta \sin \phi) \sin \frac{1\pi}{d} (x_{g} + a \sin \theta \cos \phi) \sin \phi \tag{15}$$

The radial boundary condition is given by Eq. (2b), where:

$$H_r \Big|_{\text{outside}} = -j \frac{2(26)}{\lambda} \Lambda \sqrt{1 - (\frac{\lambda}{26})^2} \sin \frac{2\pi}{6} (y_g + a \sin \theta \sin \phi)$$
 $r = a$

 $\cos \frac{1\pi}{d} (x_s + a \sin \theta \cos \phi) \sin \phi$

+ jA
$$\cos \frac{2\pi}{\delta} (y_g + a \sin \theta \sin \phi) \sin \frac{2\pi}{d} (x_g + a \sin \theta \cos \phi) \cos \phi$$
 (16)

$$H_{r} \Big|_{\substack{\text{inside} \\ r = a}} = \left[1 + K \sin^{2} \theta \right] \left(\frac{\partial \psi}{\partial r} \right)_{a} + \frac{K}{a} \sin \theta \cos \theta \left(\frac{\partial \psi}{\partial \theta} \right)_{a} - \frac{i\nu}{a} \left(\frac{\partial \psi}{\partial \phi} \right)_{a}$$
(17)

with

$$\left(\frac{\partial \psi}{\partial r}\right)_{a} = 2\eta_{1} a \cos 2\phi \sin^{2} \theta + j\eta_{2} 2a \sin 2\phi \sin^{2} \theta \qquad (18a)$$

$$\left(\frac{\partial \psi}{\partial \theta}\right)_{a} = 2\eta_{1} a^{2} \sin \theta \cos \theta \cos 2\phi + j2\eta_{2} a^{2} \sin 2\phi \sin \theta \cos \theta \qquad (18b)$$

$$\left(\frac{\partial \psi}{\partial \phi}\right)_{a} = -2\eta_{1} a^{2} \sin^{2} \theta \sin 2\phi + j2\eta_{2} a^{2} \sin^{2} \theta \cos 2\phi \tag{18c}$$

When Eq. (18) is substituted into Eq. (17), the second boundary condition equation is given by:

$$(1 + K \sin^{2}\theta) \frac{\partial \psi}{\partial r}|_{a} + \frac{K}{a} \sin\theta \cos\theta \frac{\partial \psi}{\partial \theta}|_{a} - \frac{j\nu}{a}(\frac{\partial \psi}{\partial \phi})_{a} =$$

$$-j \frac{2(2\delta)}{\lambda} A \sqrt{1 - (\frac{\lambda}{2\delta})^{2}} \sin\frac{2\pi}{\delta}(y_{g} + a \sin\theta \sin\phi)$$

$$\cos\frac{1\pi}{d}(x_{g} + a \sin\theta \cos\phi) \sin\phi$$

$$+ jA \cos\frac{2\pi}{\delta}(y_{g} + a \sin\theta \cos\phi) \sin\frac{1\pi}{d}(x_{g} + a \sin\theta \cos\phi) \cos\phi \qquad (19)$$

Equations (19) and (15) must be solved simultaneously to find x_g and y_g , the distance of the sphere from a corner of the cavity. Hence x_g and y_g give the location in the cavity that will produce the desired (2, 2, 0) mode of oscillation.

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